[8] J. Siegl, "Numerische Berechnung der Grundwelle und der höheren Wellentypen auf einer Schlitzleitung mit rechteckigem Schirm und endlicher Metallisierungsdicke." Thesis Techn. Universität Berlin, 1978.
[9] J. A. Garcia, "A wide-band quadrature hybrid coupler," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 660–661, July 1971.

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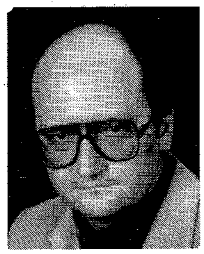

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A Synthesis Procedure for Designing 90° Directional Couplers with a Large Number of Sections

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***Abstract*—A synthesis method for designing symmetrical directional couplers with an arbitrary realizable coupling is described. The method is based on a transformation of the coupling to the time domain, where the couplings of the different sections are easily identified. The fundamental operation of the method is the Fast Fourier Transform, which makes it very efficient in computer time and memory requirements. The method is illustrated by the design of a 8.34±0.3-dB equal-ripple coupler with 201 sections.**

I. Introduction

THE DESIGN OF multisection directional couplers is usually based on an equivalent stepped impedance filter, where the reflected power is equivalent to the coupling [1]. The problem then consists of two parts. The first part is the approximation, which gives the magnitude of a realizable reflection coefficient for the equivalent stepped impedance filter. The specified reflection coefficient is optimum with respect to some requirement (for example, equal ripple).

The second part is the synthesis, where the impedances are computed from the magnitude of the reflection coefficient. The most well-known synthesis procedure is the insertion-loss method [2]. The main problem with this method is the computation of the poles and the zeros of the reflection coefficient. It has been found that the poles and the zeros cannot be computed for couplers with more than 17 to 19 sections because of the limited accuracy of the iterative procedure in this step [3].

Another synthesis method is a straight-forward optimization. Unfortunately, an optimization has several problems. It puts very heavy demands on the computer, and the

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user must enter reasonable starting values for the impedances, which makes the method difficult to use.

The purpose of this paper is to demonstrate a novel synthesis method which is easy to use and quite effective in the use of computer resources. The method is based on a Fourier transformation of the reflection coefficient to the time domain. The advantage of this approach is the separation in the time domain of the reflections from the different sections. The impedances of the stepped impedance filter can then easily be computed by a simple algorithm which is mainly concerned with the multiple reflections [4]. The main problem with this method is the computation of the phase of the reflection coefficient, which must be known before the reflection coefficient can be transformed to the time domain. This problem has been solved by a modification of the Wiener–Lee transform [5]. This modification makes it possible to compute the phase by using the Fast Fourier Transform. A very efficient method is thus assured.

II. Theory

The input data to the synthesis procedure is the magnitude of the reflection coefficient for the equivalent stepped impedance filter. It is assumed that the magnitude represents a realizable reflection coefficient.

The next step is then the computation of the phase of the reflection coefficient. There are several methods for finding the phase of a minimum phase function. The most suitable for our purpose is the Wiener–Lee transform [5].

Consider a complex function $H(\theta)$ which is minimum phase and periodic in terms of the electrical length θ. $H(\theta)$ can then be expressed as

$$H(\theta)=e^{-\alpha(\theta)-j\varphi(\theta)}. \tag{1}$$

The magnitude and phase can then be separated by taking the logarithm of $H(\theta)$

$$\alpha(\theta)+j\varphi(\theta)=-ln(H(\theta)). \tag{2}$$

$\alpha(\theta)$, which is easily computed from the magnitude of $H(\theta)$, is an even function and it is periodic. It is then possible to write $\alpha(\theta)$ as a cosine series

$$\alpha(\theta)=\sum_{n=0}^{\infty} a_n\cos(2n\theta) \tag{3}$$

$$a_n=\frac{2}{\pi}\int_{-\pi/2}^{\pi/2}\alpha(\theta)\cdot\cos(2n\theta)\cdot d\theta. \tag{4}$$

The Wiener–Lee transform then gives the phase of $H(\theta)$

$$\varphi(\theta)=-\sum_{n=1}^{\infty} a_n\sin 2n\theta. \tag{5}$$

There are, however, a few additional problems before the Wiener–Lee transform can be applied to this problem.

The first problem is the behavior of the reflection coefficient at zero frequency. For a stepped impedance filter with equal termination, the magnitude is zero at zero frequency. $\alpha(\theta)$ is then infinite, which gives problems with the cosine series ((3)). This problem can be circumvented by considering the phase of the transmitted signal. This is a useful approach, since the reflected and transmitted signals of a symmetrical stepped impedance filter are 90° out of phase. If the phase of one signal is known, the phase of the other signal is therefore easily calculated.

The transmission phase, as computed by (5), will not be total phase shift. It will only be an excess phase shift, that is, the extra phase shift compared with an equal length transmission line with the same impedance as the terminations. This fact is based on the observation that for a stepped impedance filter with all impedances equal to the terminations, $\alpha(\theta)$ will be zero since the magnitude of the transmission is one. The phase, as calculated by (5), will then also be zero, even though the phase shift is of course dependent of the length of the filter.

The phase of the reflection coefficient of the stepped impedance filter can thus be found by adding 3 components

$$\varphi(\theta)=\varphi_{\mathrm{WL}}(\theta)+N\cdot\theta-\frac{\pi}{2} \tag{6}$$

where θ is the electrical length of the sections, N is the number of sections, $\varphi_{\mathrm{WL}}(\theta)$ is the phase of the transmission computed by the Wiener–Lee transform and, $\varphi(\theta)$ is the phase of the reflection coefficient.

The Wiener–Lee transform, as stated by (3)–(5), is not suitable for computer use, because of the infinite summation and the integral in (4). The cosine series is therefore truncated. It is then possible to write the series as a Discrete Fourier Transform, since $\alpha(\theta)$ is an even and real function

$$\alpha(\theta_k)\approx\sum_{n=0}^{M-1} a_n\cos(2n\cdot\theta_k)=\sum_{n=0}^{M-1} a_n\cdot e^{j2\pi nk/M},$$
$$k=0, M-1, \qquad \theta_k=\pi k/M \tag{7}$$

where M is the number of terms in the truncated series and θ_k is the electrical length at discrete frequencies.

The advantage of using (7) is that the Discrete Fourier Transform can be used for finding the coefficients a_n instead of the integral defined in (4).

The coefficients a_n is thus found by an Inverse Fourier Transform

$$a_n\approx\frac{1}{M}\sum_{k=0}^{M-1}\alpha(\theta_k)e^{-j2\pi nk/M}, \qquad n=0, M-1. \tag{8}$$

The excess transmission phase can also be expressed as a Discrete Fourier Transform

$$\varphi_{\mathrm{WL}}(\theta_k)\approx-\sum_{n=1}^{M-1} a_n\cdot\sin(2n\theta_k)=\sum_{n=1}^{M-1} ja'_n e^{j2\pi nk/M},$$
$$k=0, M-1, a'_n=\begin{cases} a_n, & n=0, M/2\\ -a_n, & n=M/2+1, M-1\end{cases}. \tag{9}$$

The main advantage of using a discrete Fourier representation is that the computation can be performed by the

Fast Fourier Transform. The result is a very fast and efficient computation of the excess transmission phase by using (8) and (9).

After the phase of the reflection coefficient has been computed, the time domain reflection is easily found by another Discrete Fourier Transform

$$\Gamma(t_k)=\frac{1}{M}\sum_{n=0}^{M-1} e^{-\alpha(\theta_n)}(\cos(\varphi(\theta_n))-j\sin(\varphi(\theta_n))) \cdot e^{j2\pi nk/M}, \qquad k=0,M-1. \quad (10)$$

The time domain reflection coefficient can therefore be found by performing three Fast Fourier Transforms. The method is therefore quite efficient in terms of both computer time and computer memory.

The advantage of using the time domain is that the reflections from the different sections of the filter are separated. A simple algorithm [4] can therefore be used, which first removes the multiple reflections and then directly finds the impedances from the reflections of the impedance steps between the sections.

This method is slightly similar to an earlier procedure [6]. The advantage of the new method is a considerably better adaption to a computer implementation since the Fast Fourier Transform is used instead of Fourier series and determinants.

III. Theoretical Results

A symmetrical directional coupler with 201 sections has been designed by this method. The coupling has been specified as 8.34 dB±0.3 dB with an optimum equal rippler performance.

The approximation has been performed by an optimization procedure described in [7]. The new synthesis method was then used to compute the impedances. The number of terms (M) which is used in the computations should be at least 4 times larger than the number of sections and at the same time consistent with the Fast Fourier Transform. In this case, the value of M has been chosen as 1024. The coupling curve of the designed coupler can be found in Fig 1. The result of an analysis of the designed coupler can be seen in Fig 2.

The accuracy of the procedure can be found by comparing the approximation with the computed reflection coefficient of the equivalent stepped impedance filter, which can be achieved by computing an error function

$$\epsilon=\sum_{n=0}^{M}\left(|\Gamma_{\text{approx}}|-|\Gamma_{\text{filter}}|\right)^2. \quad (11)$$

The computed value of the error function is 10^{-24}.

The efficiency of the method can be judged by comparing the computer time for the approximation and the synthesis. The approximation is a nonlinear optimization with 201 variables, which requires a considerable amount of computer resources. The synthesis, on the other hand, takes approximately 3 percent of the computer time and 10 percent of the computer memory compared with the approximation.

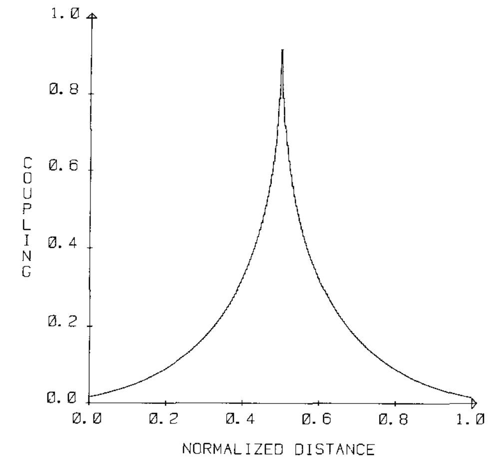


Fig. 1. Coupling factor for the designed 8.34-dB coupler.

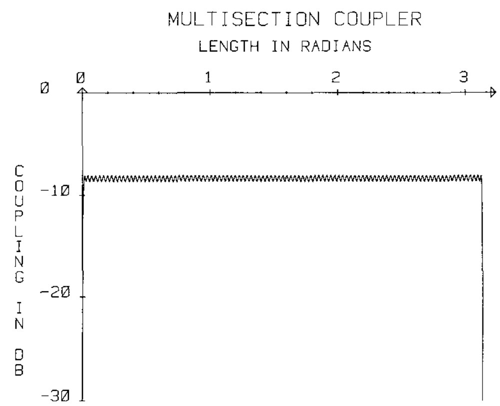


Fig. 2. Theoretical performance of the designed 8.34-dB coupler.

IV. Conclusions

A very efficient method of synthesizing multisection directional couplers has been presented. An arbitrary realizable coupling can be synthesized efficiently, since the main computation involves the Fast Fourier Transform. The computing time is a few percent of a standard optimization procedure. Theoretical results for a 201 section coupler have been included.

Acknowledgment

The help and encouragement of Prof. E. F. Bolinder is gratefully acknowledged.

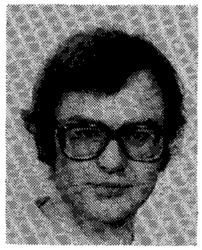
References

[1] L. Young, "The analytical equivalence of TEM-mode directional couplers and transmission line stepped impedance filters," in *Proc. Instit. Elect. Eng.*, vol. 110, Feb. 1963, pp. 275–281.

[2] E. G Cristal and L. Young, "Theory and tables of optimum symmetrical TEM-mode coupled transmission line directional couplers," *IEEE Trans. Microwave Theory Tech*, vol. MTT-13, pp. 544–558, Sept. 1965.

[3] F. Arndt and M. Hoffmann, "Äqui-Extremwerld-Polynome und synthese von 90° kopplern," *Arch. Elek. Ubertragung*, vol. AEU-26, pp. 105–110, Mar. 1972

[4] N. D. Megill, "TDR's profile impedances of backplanes and pc boards," *Electronics*, vol. 59, pp. 113–117, July 14, 1981.
[5] A. Papoulis, *The Fourier Integral and its Application*. New York: McGraw-Hill, 1962, pp. 201–210.
[6] J. D. Rhodes, "Fourier coefficient design of stepped impedance transmission line networks," *Int. J. Circuit Theory Appl.*, vol. 1, pp. 363–371, 1973.
[7] G. Saulich, "A new approach in the computation of ultra-high degree equal ripple polynomials for 90°-coupler synthesis," *IEEE Trans. Microwave Theory Tech*, vol. MTT-29, pp. 132–135, Feb. 1981.



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The Use of Symmetry to Simplify the Integral Equation Method with Application to 6-Sided Circulator Resonators

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Abstract—**In this paper it is shown that for planar two-dimensional problems with symmetry, the dimensions of the matrices, which must be inverted to obtain a solution using the integral equation method, can be substantially reduced. For instance, for a three-fold symmetric hexagonal circulator junction with N segments about the periphery, the dimension of matrices to be inverted is reduced to $N/3$ from the usual N. It is demonstrated that for six-sided resonators with three-fold symmetry, a very good approximation to the equivalent admittance can be obtained with only 12 segments around the periphery, meaning that only 4×4 matrices need be inverted.**

I. Introduction

One of the most general methods for analyzing arbitrarily shaped planar circuits is based on the contour integral representation of the wave equation as presented by Okoshi and Miyoshi about 1970 [1], [2]. Initially an isotropic dielectric was assumed. In 1977 the theory was extended to include nonreciprocal circuits which use ferrites magnetized perpendicular to the conducting planes [3]. A drawback of the contour integral method is the relatively long computational times required for analysis since, for complex circuit patterns, large-dimensional matrices must be inverted. Some methods are available for improving the computational efficiency for circuits with special properties. One method is based on the observation that the Green's function suitable to the integral equation method is not unique [4]. The arbitrariness of the Green's function can be used to optimize the accuracy of the numerical results, or equivalently decrease the computational time necessary to obtain a given accuracy. The second method takes advantage of symmetries in the planar circuit. In the design of a 3-dB hybrid with two symmetry planes Okoshi, Imai, and Ito computed the reflection coefficient from one of the four congruent quarter circuits for each of the four eigenexcitations of the entire circuit [5]. The scattering parameters of the hybrid are then given as a linear combination of these scattering matrix eigenvalues. In this paper it will be shown in general how existing symmetries of a junction may be used to simplify the diagonalization of matrices. The computational effort is reduced by a factor comparable with the order of symmetry of the circuit.

Though planar junction, three-port circulators with six-sided resonators constitute a natural application of the integral equation method, and these circulators have been built commercially for many years now, the method has with one exception [6] not been applied to their analysis. Consequently, it was felt to be worthwhile to combine a

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